

Five-parton amplitudes with two quarks and two photons at next-to-leading order

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We discuss one-loop five-parton amplitudes with two-quark, two-photon, or three-photon external legs. The amplitudes are required to evaluate the NLO corrections for the $\gamma\gamma$ jet production process at hadron colliders. The results have been already discussed by several groups in terms of QCD amplitudes. Here, we present a more straightforward version of one-loop calculations without using the QCD amplitudes.

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I. INTRODUCTION

The search for the Higgs boson will be one of the most important issues of the CERN Large Hadron Collider (LHC) experiments. However, if the mass of the Higgs boson is light ($M_H \leq 140$ GeV), the Higgs boson search at the LHC will not be so easy. In this range, the rare decay mode into two photons ($H \rightarrow \gamma\gamma$) is expected to give rather clear signals [1]. Many calculations have been done for this process, including radiative corrections [2,3]. They found that the next-to-leading order (NLO) corrections to Higgs production are very large [2]. The next-to-next-to-leading order (NNLO) corrections also have been evaluated in the large top quark mass limit [3]. The NNLO corrections show a good convergence in the perturbative expansion. Now, the uncertainty of the higher order corrections for the Higgs production signal is less serious. On the other hand, the QCD background still has large uncertainties. The NLO corrections of the QCD background ($q\bar{q} \rightarrow \gamma\gamma$) are also very large. To make matters worse, NNLO corrections of gluon-gluon initiated processes, such as the box type correction $gg \rightarrow \gamma\gamma$, may be very large due to the large gluon distribution [4,5]. Actually, the size of a NNLO correction $gg \rightarrow \gamma\gamma$ is comparable to the LO contribution $q\bar{q} \rightarrow \gamma\gamma$. Thus, at least, full NNLO analysis will be required to obtain the significant evaluation. However, the NNLO analysis of the QCD backgrounds is still on the way [6,7].

It has been pointed out that we can improve the signal-to-background ratio by considering the associated production of the Higgs boson with a high transverse energy jet ($pp \rightarrow H \text{ jet} \rightarrow \gamma\gamma \text{ jet}$) [8]. The existence of a high P_T jet in the final state allows us to choose suitable cuts to suppress the QCD background [8]. In addition, for this process, the ambiguity of the higher order corrections is less serious. The subprocess with two-gluon initial states $gg \rightarrow g\gamma\gamma$ again contributes first in the NNLO, which is believed to dominate the NNLO contributions. However, the LO correction, in which the main contribution comes from the subprocess $q\bar{q} \rightarrow q\gamma\gamma$, is sufficiently larger than the NNLO contributions because of the large gluon distribution [4]. Thus we expect that the NLO analysis is fairly accurate to understand the QCD background.

To evaluate the NLO corrections of the QCD background, we need the tree-level QCD amplitudes for six partons ($q\bar{q}\gamma\gamma gg, q\bar{q}Q\bar{Q}\gamma\gamma$) and one-loop amplitudes for the five

partons ($q\bar{q}\gamma\gamma g$). In Ref. [10], they present the compact expressions for the six-parton tree-level amplitudes. The one-loop amplitudes are also discussed in Refs. [9,10]. They provided the systematic procedure to express the one-loop five-parton amplitudes with using the known results of the $q\bar{q}ggg$ amplitudes or “primitive amplitudes” [11]. However, the QCD amplitudes include many unnecessary terms that disappear from the final expression of multiphoton amplitudes. Thus, their one-loop expressions are still rather complicated. In this paper, we carry out the direct calculations of the one-loop five-parton amplitudes, which are involving two massless quarks, two or three external photons. Our final goal of this paper is to obtain the explicit expressions for the one-loop $q\bar{q}g\gamma\gamma$ amplitudes without using QCD amplitudes.

The paper is organized as follows. It is now popular that QCD matrix elements of multi-partons are expressed in terms of the color ordered helicity amplitudes. In Sec. II, we explain some technical prescriptions of these methods. We also explain known general forms of tree-level amplitudes with external gluons and/or photons. In Sec. III, we present one-loop helicity amplitudes for five-parton processes including two or three external photons. In Sec. IV, we give some concluding remarks.

II. TREE-LEVEL AMPLITUDES

The color decomposition [12] technique and helicity basis method [13] are standard techniques to express the multiparton amplitudes. The color decomposition method is the procedure that constructs color ordered gauge invariant partial amplitudes in the QCD. At the tree level, amplitudes $\mathcal{A}_n$ involving two quarks and $n-2$ gluons can be decomposed into the partial amplitudes $\mathcal{M}$'s, which are characterized by the single string of the group matrices [14],

$$\mathcal{A}_n = g^{n-2} \sum_{a_i \in S_{n-2}} (T^{a_3} \cdots T^{a_n}) A_n(q^i, \bar{q}^i, g^i, \dots, g^i),$$

where i_j are parton helicities. T^a ($a=1,2,\dots,N^2-1$) are the matrices of the gauge group in the fundamental representation. S_n denotes the set of noncyclic permutations over $1, \dots, n$.

We also introduce the spinor helicity basis method [13]. We use the popular notations for the helicity basis of a massless spinor field ψ [14],

$$\langle q^\pm | \equiv \frac{1}{2} \bar{\psi}(q)(1 \mp \gamma_5), \quad |q^\pm \rangle \equiv \frac{1}{2}(1 \pm \gamma_5)\psi(q).$$

In this paper, all external momenta are taken to be outgoing. All color ordered helicity amplitudes are expressed in terms of following spinor products:

$$\langle pq \rangle \equiv \langle p^- | q^+ \rangle, \quad [pq] \equiv \langle p^+ | q^- \rangle,$$

$$[pq]\langle qp \rangle = s_{pq} \equiv 2p \cdot q.$$

We can also describe external gauge fields by using the spinor helicity basis. The polarization vectors can be written in terms of massless spinors $|p^\pm \rangle$ and $|k^\pm \rangle$,

$$\varepsilon^\pm(p, k) = \pm \frac{\langle p^\pm | \gamma_\mu | k^\pm \rangle}{\sqrt{2} \langle k^\mp | p^\pm \rangle}, \quad (1)$$

where p is the gauge boson momentum and k is the arbitrary momentum that satisfies $k^2 = 0$. We call this momentum k as the *reference* momentum. Physical quantities do not depend on the *reference* momentum, because a change in the *reference* momentum is equivalent to a gauge transformation:

$$\varepsilon^+(p, k)_\mu \rightarrow \varepsilon^+(p, k')_\mu - \sqrt{2} \frac{\langle kk' \rangle}{\langle kp \rangle \langle k' p \rangle} p_\mu.$$

This means that we can choose an appropriate *reference* momentum for any gauge invariant subset of the full amplitude. For example, in this formula, we obtain the following identities:

$$\not{\varepsilon}^\pm(p, k) |k^\pm \rangle = 0, \quad \langle k^\mp | \not{\varepsilon}^\pm(p, k) = 0. \quad (2)$$

Using these identities, we can easily show that the amplitudes in which all gluons have the same helicity vanish [14],

$$A_n^{tree}(\bar{q}, q, g_3^+, g_4^+, \dots, g_n^+) = 0. \quad (3)$$

From the above formula, we also obtain the simple expression of the amplitudes in which one gluon has negative helicity and all other gluons have positive helicity [10],

$$A_n^{tree}(\bar{q}_1^+, q_2^-, g_3^+, \dots, g_j^-, \dots, g_n^+) = i \frac{\langle 1j \rangle \langle 2j \rangle^3}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}, \quad (4)$$

where the j th gluon has the negative helicity. Here, we followed the notations and conventions given in Ref. [14]. (In Sec. III, we use the different normalization condition of group factor T^a .)

The amplitudes involving external photons are obtained by summing over permutations of gluon matrix elements [14]. The amplitudes containing gluons and photons with maximally helicity-violating (MHV) configuration $(-, -, +, \dots, +)$ are [10]

$$A_n^{tree}(\bar{q}_1^+, q_2^-, g_3, \dots, g_{r+2}, \gamma_{r+3}, \dots, \gamma_{r+m+2}) = i \frac{\langle 1i \rangle \langle 2i \rangle^3}{\langle 12 \rangle \langle 23 \rangle \dots \langle (r+2)1 \rangle} \prod_{j=r+3}^{r+m+2} \left[\frac{\langle 21 \rangle}{\langle 2j \rangle \langle j1 \rangle} \right], \quad (5)$$

where the i th gluon or photon has the negative helicity. To obtain the one-loop level photons amplitudes, similar procedure have been applied to the one-loop amplitudes [9,10].

The amplitudes with other configuration of helicities are obtained by Parity inversion and charge conjugation [11]. Parity inversion reverses the sign of all helicities of external legs. This conversion is achieved by taking the complex conjugation. In terms of the helicity basis method, this operation is equivalent with the replacement of spinor products $\langle ij \rangle \leftrightarrow [ji]$, but with no substitution of $i \rightarrow -i$. In addition, a factor -1 is required for each quark antiquark pair. Charge conjugation replaces quarks and antiquarks without changing helicities. From these relations, we can reduce the number of independent partial amplitudes. For the case of $q\bar{q}g\gamma\gamma$ amplitude, we only need to calculate the amplitudes with following three types of helicity configurations: $(\bar{q}^-, q^+, g^+, \gamma^+, \gamma^+)$ and $(\bar{q}^-, q^+, g^\pm, \gamma^\mp, \gamma^+)$.

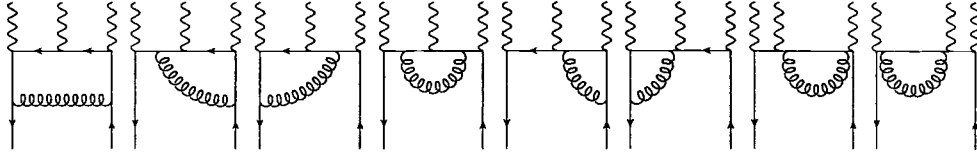
III. ONE-LOOP RESULTS

The one-loop $q\bar{q}\gamma\gamma g$ amplitudes can be written by the three types gauge independent partial amplitudes $\mathcal{M}_5^i(i=1,2,3)$,

$$\begin{aligned} \mathcal{A}_5^{1-loop}(q\bar{q}\gamma\gamma g) = & -\frac{e_q^2 g^3 T^a}{2} \left\{ \frac{1}{N_c} \mathcal{M}_5^1(q\bar{q}\gamma\gamma g) \right. \\ & \left. + N_c \mathcal{M}_5^2(q\bar{q}\gamma\gamma g) \right\} \\ & - \sum_{i=1}^{n_f} \frac{e_{q_i}^2 g^3 T^a}{2} \mathcal{M}_5^3(q\bar{q}\gamma\gamma g). \quad (6) \end{aligned}$$

$\mathcal{M}_5^1$ and $\mathcal{M}_5^2$ have different dependence on the color factor $N_c (=3$ for QCD) and $\mathcal{M}_5^3$ is the fermion loop contribution. e_q is the charge of the external quarks. e_{q_i} are the charges of loop fermions and n_f is flavor. To explain the one-loop results, we change the normalization condition of the group generators as $\text{Tr}(T^a T^b) = \delta^{ab}/2$.

To carry out the one-loop calculations, we need the information on the Feynman integrals. We follow the technology of the Feynman integral calculation which was discussed in Refs. [15,16]. We use dimensionally regulated one-loop integrals in $4-2\epsilon$ dimensions. In the conventional dimensional regularization (CDR) scheme, both momentum components and helicity states are dealt with in $D(=4-2\epsilon)$ dimensions [17]. Thus, all gluons and photons have $2-2\epsilon$ helicity states. On the other hand, the spinor helicity basis is defined in four dimensions. To apply the helicity basis method in the one-loop calculations, we need some modification in the regularization scheme. The 't Hooft-Veltman scheme is one of the solutions where polarization of ob-

FIG. 1. Feynman diagrams for $\mathcal{O}(1/N_c)$ contributions.

served particles is dealt with in four dimensions (then observed gluons and photons have 2 helicity state). The four-dimensional helicity (FDH) scheme [17] is a more efficient approach when using the spinor helicity method. In this scheme, momentum components of unobserved particles are dealt with in $4 - 2\epsilon$ dimensions, but we treat all helicities in four dimensions. Thus, both observed and unobserved gauge bosons have 2 helicity states.

A. $\mathcal{M}_5^1(\bar{q}qg\gamma\gamma)$

We give the Feynman diagrams which contribute to the partial amplitudes $\mathcal{M}_5^1$ in Fig. 1. Here, one of the external gauge boson legs is a gluon and others are photons. We notice that the amplitudes $\mathcal{M}_5^1$ are obtained from the two-quark three-photon amplitudes $\mathcal{A}(q\bar{q}\gamma\gamma\gamma)$ by converting one of the photons into a gluon.

$$\mathcal{M}_5^1(q, \bar{q}, g, \gamma, \gamma) = m_5(q, \bar{q}, \gamma \rightarrow g, \gamma, \gamma),$$

$$\mathcal{A}^{1-loop}(q, \bar{q}, \gamma, \gamma, \gamma) = e_q^3 g^2 \frac{N_c^2 - 1}{2N_c} m_5(q, \bar{q}, \gamma, \gamma, \gamma).$$

Therefore, we present the $q\bar{q}\gamma\gamma\gamma$ amplitudes at first.

From the analogy of Eq. (2) and Eq. (3), the tree level amplitudes in which all of the photons have the same helicity vanish:

$$m_n^{tree}(\bar{q}, q, \gamma_3^+, \gamma_4^+, \dots, \gamma_n^+) = 0.$$

This result ensures that the corresponding amplitudes at the one-loop level must be infrared and ultraviolet finite. The explicit form of the one-loop amplitude for the helicity $(q_1^+, \bar{q}_2^-, \gamma_3^+, \gamma_4^+, \gamma_5^+)$ is

$$\begin{aligned} m_5(q_1^+ q_2^- \gamma_3^+ \gamma_4^+ \gamma_5^+) = & \frac{i}{16\pi^2} \frac{2\sqrt{2}}{[24]\langle 14\rangle\langle 35\rangle\langle 45\rangle\langle 34\rangle} \left[-\frac{[13][24]\langle 14\rangle^2\langle 25\rangle\langle 23\rangle}{\langle 24\rangle\langle 51\rangle} + \frac{s_{24}\langle 35\rangle[51]\langle 12\rangle}{\langle 23\rangle} - \frac{s_{24}s_{45}\langle 35\rangle\langle 12\rangle}{\langle 23\rangle\langle 51\rangle} \right. \\ & + [24][51]\langle 12\rangle\langle 45\rangle + \frac{s_{24}[51]\langle 25\rangle\langle 13\rangle}{\langle 23\rangle} - 2\frac{s_{13}[24]\langle 12\rangle\langle 45\rangle}{\langle 51\rangle} + \frac{[24]\langle 12\rangle\langle 45\rangle s_{23}}{\langle 51\rangle} - \frac{[24]\langle 12\rangle\langle 45\rangle s_{12}}{\langle 51\rangle} \Big] \\ & + (3 \leftrightarrow 5). \end{aligned}$$

This result is consistent with the result of Ref. [10].

An independent nonvanishing tree-level partial amplitude m_5^{tree} is given by the helicity configuration $(q^-, \bar{q}^+, \gamma^-, \gamma^+, \gamma^+)$,

$$m_5^{tree}(q_1^+ \bar{q}_2^- \gamma_3^+ \gamma_4^- \gamma_5^+) = \frac{i2\sqrt{2}\langle 12\rangle\langle 24\rangle^2}{\langle 25\rangle\langle 23\rangle\langle 51\rangle\langle 13\rangle}. \quad (7)$$

Here, we define the partial amplitudes m_5 's as

$$A_5^{tree}(q\bar{q}\gamma\gamma\gamma) \equiv e^3 m_5^{tree}(q\bar{q}\gamma\gamma\gamma).$$

The difference between Eq. (5) and Eq. (7) in the factor $2\sqrt{2}$ comes from the different normalization of group factor T^a . The corresponding one-loop level partial amplitude pos-

sesses ultraviolet and infrared divergences. It is well known that the singular part of one-loop n -point amplitudes have the universal structure [18,19],

$$m_n^{1-loop}|_{\text{singular}} = c_\Gamma m_n^{tree} \left[-\frac{1}{\epsilon^2} \sum_{j=1}^n \mathcal{S}_j^{[n]} \left(\frac{\mu^2}{-s_{j,j+1}} \right)^\epsilon + \mathcal{C}^{[n]} \frac{1}{\epsilon} \right], \quad (8)$$

where

$$c_\Gamma \equiv \frac{(4\pi)^\epsilon \Gamma(1+\epsilon) \Gamma^2(1-\epsilon)}{16\pi^2 \Gamma(1-2\epsilon)}$$

with $D = 4 - 2\epsilon$ and μ is the renormalization scale. m^{tree} is the tree-level partial amplitude. $\mathcal{S}_j^{[n]}$ are the coefficients of the soft singularities and $\mathcal{C}_j^{[n]}$ are the sum of the coefficients

of the collinear and ultraviolet singularities which depend on the particle contents of amplitudes. For the massless QCD, accurate values of the coefficients $\mathcal{S}_j^{[n]}$ and $\mathcal{C}_j^{[n]}$ are well known [11,19]. Because of this fact, it is easy to check the singular part of one-loop level amplitudes. We will discuss this matter again in the end of this section. Here, we present the amplitudes in which ultraviolet divergence is unsubtracted.

We also introduce the following functions to explain the nonsingular parts:

$$L(s, t, u) \equiv \text{Li}_2\left(1 - \frac{s}{u}\right) + \text{Li}_2\left(1 - \frac{t}{u}\right) + \ln \frac{s}{u} \ln \frac{t}{u} - \frac{\pi^2}{6}$$

$$R \ln^n\left(\frac{s}{t}\right) \equiv \frac{\ln^n\left(\frac{s}{t}\right)}{(s-t)^n},$$

where $\text{Li}_2(Z)$ is the dilogarithm function [20],

$$\text{Li}_2(Z) \equiv - \int_0^Z \frac{dx}{x} \log(1-x).$$

The explicit form of the one-loop partial amplitude for the helicity configuration $(q^-, \bar{q}^+, \gamma^-, \gamma^+, \gamma^+)$ is

$$\begin{aligned} m_5(q_1^+ q_2^- \gamma_3^+ \gamma_4^- \gamma_5^+) = c_1 m^{\text{tree}} & \left\{ -\frac{2}{\epsilon^2} \left(\frac{\mu^2}{-s_{12}} \right)^\epsilon - \frac{3}{\epsilon(1-2\epsilon)} \left(\frac{\mu^2}{-s_{12}} \right)^\epsilon - 3 - \delta_D \right\} + \frac{i}{(4\pi)^2} \sqrt{\frac{2}{[24]^2 \langle 35 \rangle^2 \langle 12 \rangle}} \left[f_1 \ln\left(\frac{s_{34}}{s_{12}}\right) \right. \\ & + f_2 \ln\left(\frac{s_{14}}{s_{45}}\right) + f_3 R \ln^1\left(\frac{s_{23}}{s_{45}}\right) + f_4 R \ln^2\left(\frac{s_{14}}{s_{23}}\right) - 4 \frac{\langle 35 \rangle [24] s_{24} \langle 12 \rangle^2 s_{12} (-[13] \langle 34 \rangle + [51] \langle 45 \rangle)}{\langle 23 \rangle \langle 51 \rangle s_{13}^2} \\ & \times \tilde{L}(s_{23}, s_{12}, s_{45}) - 4 \frac{\langle 45 \rangle^2 \langle 12 \rangle^3 [24]^2 \langle 35 \rangle}{\langle 51 \rangle^3 \langle 23 \rangle} \left(\tilde{L}(s_{14}, s_{45}, s_{23}) + \frac{s_{51}^2}{s_{13}^2} \tilde{L}(s_{23}, s_{12}, s_{45}) \right) \\ & - 4 \frac{\langle 12 \rangle \langle 35 \rangle s_{24}^2}{\langle 51 \rangle \langle 23 \rangle} (L(s_{12}, s_{51}, s_{34}) + L(s_{34}, s_{24}, s_{51})) \\ & + \left\{ 2 \frac{[35](-[14] \langle 51 \rangle \langle 34 \rangle + \langle 23 \rangle [24] \langle 45 \rangle)(s_{34} + s_{45})}{s_{35}} - 2 \frac{s_{34}^2 (-s_{12} + s_{34} - 2s_{51})}{s_{51}} \right. \\ & - 2 \frac{[23]([51] \langle 12 \rangle \langle 35 \rangle - \langle 34 \rangle [45] \langle 25 \rangle)(s_{51} - s_{34})^2}{s_{23} s_{51}} - 2 \langle 34 \rangle [45] \langle 51 \rangle [13] + 2 \langle 23 \rangle [24] \langle 45 \rangle [35] \\ & \left. \left. + 2 s_{23} s_{12} - 4 s_{12} (s_{51} - s_{34}) \right\} L(s_{45}, s_{34}, s_{12}) + R_0 + (3 \leftrightarrow 5) \right]. \end{aligned}$$

The scheme dependence is controlled by the parameter δ_D . We obtain $\delta_D=0$ in the FDH scheme and $\delta_D=1$ in the 't Hooft-Veltman scheme. Here, we also used the following definitions:

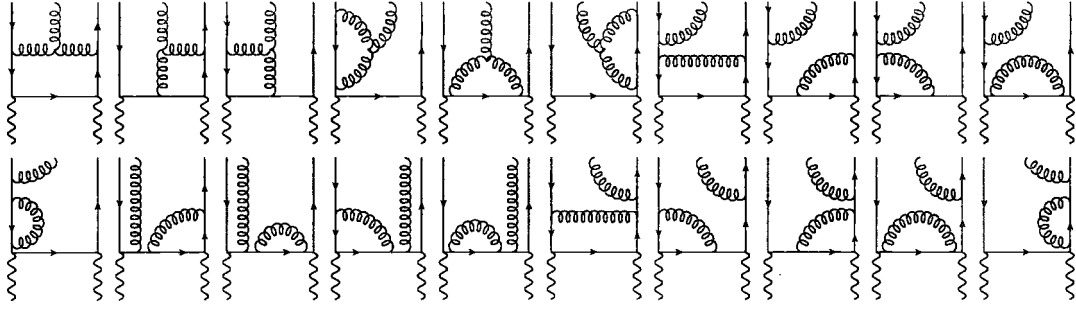
$$\tilde{L}(s_{23}, s_{12}, s_{45}) \equiv L(s_{23}, s_{12}, s_{45}) - s_{13} R \ln^1\left(\frac{s_{23}}{s_{45}}\right)$$

$$\tilde{L}(s_{14}, s_{45}, s_{23}) \equiv L(s_{14}, s_{45}, s_{23}) - s_{51} R \ln^1\left(\frac{s_{23}}{s_{45}}\right) - s_{51} R \ln^1\left(\frac{s_{14}}{s_{23}}\right).$$

f_i ($i=1, \dots, 4$) and R_0 are given by

$$\begin{aligned} f_1 = & 6 \frac{\langle 12 \rangle \langle 35 \rangle s_{45}^2}{\langle 13 \rangle \langle 25 \rangle} - \frac{\langle 45 \rangle [14] \langle 12 \rangle (3 s_{45} + 2 s_{23})}{\langle 25 \rangle} - 2 \frac{\langle 12 \rangle [25] \langle 35 \rangle (4 s_{45} + s_{25})}{\langle 13 \rangle} - 4 \frac{\langle 12 \rangle [23] \langle 35 \rangle (s_{34} + s_{23})}{\langle 51 \rangle} \\ & - 2 \frac{[45] \langle 34 \rangle \langle 25 \rangle (2 s_{12} + s_{34} - s_{51})}{\langle 23 \rangle} - 2 \frac{\langle 45 \rangle [51] \langle 12 \rangle [24] s_{23}}{s_{25}} + 2 \frac{s_{23} s_{12} s_{45}}{s_{25}} + 3 \frac{s_{45}^2 s_{12}}{s_{25}} - 3 \frac{\langle 45 \rangle [51] \langle 12 \rangle [24] s_{45}}{s_{25}} \\ & - 2 s_{25} s_{34} + 2 s_{25} s_{12} + 2 s_{25}^2 + 8 s_{25} s_{45} + 4 s_{23} s_{12} + 6 s_{45}^2 + 12 s_{12} s_{45} \end{aligned}$$

$$\begin{aligned}
f_2 &= 2 \frac{[13]\langle 23 \rangle \langle 51 \rangle (s_{23} + 3 s_{12})}{\langle 25 \rangle} + 4 \frac{s_{12} \langle 35 \rangle [45] \langle 24 \rangle}{\langle 23 \rangle} \\
f_3 &= -2 \frac{[51]\langle 12 \rangle [23] \langle 35 \rangle (s_{23} + s_{34}) (s_{23} s_{12} + s_{23} s_{13} + s_{12} s_{34} + 3 s_{34} s_{13})}{s_{23} s_{51}} \\
&\quad - 2 \frac{[51]\langle 12 \rangle [23] \langle 35 \rangle (-2 s_{12} s_{51} - 5 s_{34} s_{45} + 4 s_{12} s_{34} + 2 s_{51} s_{45})}{s_{23}} - 2 \langle 12 \rangle [24] [13] \langle 34 \rangle (3 s_{34} - 2 s_{45} + 2 s_{12}) \\
&\quad + 10 s_{34}^2 s_{12} - 4 s_{12} s_{34} s_{51} + 10 s_{12} s_{23} s_{34} - 6 s_{23} s_{12}^2 - 4 s_{23} s_{12} s_{13} - 4 s_{12} s_{51} s_{23} - 4 s_{12} s_{51} s_{13} - 6 s_{12}^2 s_{34} - 6 s_{12} s_{34} s_{13} \\
&\quad + 2 s_{23} s_{13}^2 \\
f_4 &= 2 \frac{[51]\langle 12 \rangle [23] \langle 35 \rangle (s_{34} - s_{51}) (s_{51} + s_{45}) (s_{34} s_{51} + 2 s_{45} s_{51} - s_{34} s_{45})}{s_{23} s_{51}} \\
&\quad - 2 \frac{[51]\langle 12 \rangle [23] \langle 35 \rangle (-s_{12} + s_{34}) (-s_{23} s_{12} + s_{34} s_{23} - 2 s_{34} s_{45})}{s_{51}} + 2 (s_{51} + s_{45})^2 (-\langle 34 \rangle [45] \langle 51 \rangle [13] + [14] [25] \langle 12 \rangle \\
&\quad \times \langle 45 \rangle - s_{12} s_{13}) + 2 (s_{34} - s_{51}) (s_{51} + s_{45}) (s_{45} s_{12} + s_{51}^2 - s_{34} s_{51} + 2 [34] \langle 45 \rangle [51] \langle 13 \rangle) + 2 (-s_{12} + s_{34}) (s_{34} - s_{51}) \\
&\quad \times (s_{12} s_{51} + 2 [51] \langle 12 \rangle [23] \langle 35 \rangle + s_{45}^2) + (-\langle 45 \rangle [51] \langle 12 \rangle [24] - [23] \langle 34 \rangle [45] \langle 25 \rangle) (s_{12} - s_{34}) (s_{12} + s_{45}) + 2 s_{23} \\
&\quad (-s_{12} + s_{34}) (s_{12}^2 + s_{45} s_{12} + 2 [34] \langle 45 \rangle [51] \langle 13 \rangle + s_{34}^2 - s_{51}^2) + 2 \langle 23 \rangle [25] [34] \langle 45 \rangle s_{23} (s_{51} + s_{45}) - s_{45} (s_{34} + 5 s_{23}) \\
&\quad (-s_{12} + s_{34}) (s_{34} - s_{51}) - (-\langle 45 \rangle [51] \langle 12 \rangle [24] - [23] \langle 34 \rangle [45] \langle 25 \rangle - s_{34} s_{51} + s_{45}^2 + s_{51}^2) s_{45} (-s_{23} + s_{34}) + (s_{34} \\
&\quad - s_{51}) (s_{51} + s_{45}) (-s_{45}^2 + 3 s_{45} s_{51} + 2 s_{45} s_{12} + 6 s_{45} s_{23} + 2 s_{23} s_{51}) + (-s_{12} + s_{34}) (s_{51} + s_{45}) (3 s_{45} s_{12} - 10 s_{45} s_{23} \\
&\quad - 2 s_{34} s_{45} - s_{12}^2 + s_{34} s_{12} - 3 s_{51}^2) + (s_{51} + s_{45}) (-2 s_{45}^2 s_{23} - s_{45} s_{51}^2 + 5 s_{45} s_{23} s_{51} + 2 s_{51}^3) - (s_{51} + s_{45})^2 s_{45}^2 + (-s_{12} \\
&\quad + s_{34}) (s_{12}^2 s_{45} - 3 s_{12} s_{45}^2 - 4 s_{12} s_{23} s_{34} + 6 s_{45}^2 s_{23} + s_{45} s_{34}^2 + s_{45} s_{23}^2) - s_{45}^2 (-s_{45}^2 - s_{45} s_{23} - s_{45} s_{51} - 3 s_{23} s_{51} \\
&\quad + 4 s_{34} s_{23}) + 4 (-s_{12} + s_{34})^2 s_{23} s_{45} - 2 s_{45} (-s_{12} + s_{34}) (s_{34} - s_{51}) (s_{51} + s_{45}) + (s_{34} - s_{51})^2 (-s_{12} + s_{34}) (s_{51} + s_{45}) \\
&\quad - s_{23} (-s_{12} + s_{34})^2 (-s_{45} + s_{12}) - 2 (s_{12} - s_{34})^2 (s_{51} + s_{45}) (s_{23} + s_{45}) + 2 s_{23} (s_{51} + s_{45})^2 (-s_{45} + s_{51} + s_{23}) + 2 s_{34} (s_{51} \\
&\quad + s_{45})^2 (s_{34} + s_{25} + s_{23}) - (-s_{12} + s_{34})^2 (s_{51} + s_{45}) s_{34} - 2 (-s_{12} + s_{34})^2 (s_{34} - s_{51}) s_{23} \\
R_0 &= \frac{[25] (\langle 45 \rangle [34] \langle 23 \rangle + \langle 51 \rangle \langle 24 \rangle [14]) (2 s_{51} - 3 s_{34} - s_{45}) s_{24}}{(s_{34} + s_{45}) s_{25}} + \frac{\langle 14 \rangle [34] (-[12] \langle 23 \rangle + [51] \langle 35 \rangle) s_{35} s_{12}}{s_{14} s_{23}} \\
&\quad + \frac{s_{35}^2 (s_{23} + s_{34})}{s_{51}} - \frac{\langle 45 \rangle [24] (-[51] \langle 12 \rangle + [35] \langle 23 \rangle) s_{35}^2}{s_{51} (s_{51} + s_{45})} - \frac{s_{35}^2 (s_{12} + s_{34} + s_{45} + s_{23})}{s_{51} + s_{45}} \\
&\quad + \frac{(-\langle 51 \rangle [12] \langle 23 \rangle [35] + [34] [51] \langle 14 \rangle \langle 35 \rangle - s_{51} s_{23}) s_{12} s_{35}}{s_{23} (s_{51} + s_{45})} \\
&\quad + 2 \frac{s_{12} s_{14} ([51] \langle 12 \rangle [23] \langle 35 \rangle - \langle 34 \rangle [45] \langle 51 \rangle [13] + s_{45}^2 - s_{23} s_{13} - 2 s_{23} s_{45})}{s_{23} (s_{34} + s_{45})} \\
&\quad + 2 \frac{((51) [25] \langle 23 \rangle [13] - [34] \langle 45 \rangle [51] \langle 13 \rangle + s_{51}^2 + s_{45} s_{51} + s_{45}^2) s_{14}}{s_{34} + s_{45}} \\
&\quad + 3 \frac{[51] (\langle 13 \rangle \langle 45 \rangle [34] + \langle 12 \rangle [23] \langle 35 \rangle) (s_{23} + s_{34})^2}{s_{51} s_{23}} - 3 \frac{(s_{12} - s_{45}) s_{34}^2}{s_{23}} \\
&\quad + \frac{[23] (\langle 25 \rangle [45] \langle 34 \rangle - [51] \langle 12 \rangle \langle 35 \rangle) (2 s_{25} + 3 s_{34})}{s_{23}} + 2 \frac{s_{45} s_{12} (s_{51} + s_{45})}{s_{23}} + 14 \langle 23 \rangle [34] \langle 45 \rangle [25] + 7 [12] \langle 23 \rangle [34] \\
&\quad \times \langle 14 \rangle - 7 [51] \langle 12 \rangle [23] \langle 35 \rangle + 7 s_{45} s_{34} + 5 s_{51}^2 - 6 s_{51} s_{23} + s_{23}^2 - 3 s_{51} s_{12} + s_{12} s_{23} - s_{23} s_{45} - 5 s_{34}^2 + 2 s_{45}^2 + s_{12}^2 \\
&\quad - 4 s_{23} s_{34} + 3 s_{45} s_{51} + 4 s_{12} s_{34} - 7 s_{45} s_{12} - s_{34} s_{51}.
\end{aligned}$$

FIG. 2. Feynman diagrams for $\mathcal{O}(N_c)$.

B. $\mathcal{M}_5^2(\bar{q}qg\gamma\gamma)$

The Feynman diagrams which contribute to the $\mathcal{O}(N_c)$ part are given in Fig. 2. The amplitudes of all helicity positive gluon and photons vanish again at the tree level. The corresponding amplitude at one-loop level is infrared and ultraviolet finite as

$$\mathcal{M}_5^2(g_1^+ \bar{q}_2^- \gamma_3^+ \gamma_4^+ q_5^+) = \frac{i}{16\pi^2} \frac{2\sqrt{2}}{[12]\langle 51\rangle\langle 14\rangle\langle 13\rangle\langle 34\rangle} \left[\frac{[12][51]\langle 13\rangle\langle 14\rangle\langle 25\rangle^2}{\langle 23\rangle\langle 45\rangle} - \frac{[12][34]\langle 13\rangle\langle 24\rangle\langle 14\rangle\langle 25\rangle}{\langle 45\rangle\langle 12\rangle} \right] + (3 \leftrightarrow 4).$$

This result is also consistent with the result of Ref. [10].

Tree-level amplitudes with the helicity configuration $(\bar{q}, q, \gamma^+, \gamma^+, g^+)$ are nonvanishing. Corresponding one-loop amplitudes have the ultraviolet and infrared divergences, again. The amplitude in which a gluon has negative helicity and two photons have positive helicity is

$$\begin{aligned} \mathcal{M}_5^2(g_1^- \bar{q}_2^- \gamma_3^+ \gamma_4^+ q_5^+) = & -c_\Gamma \mathcal{M}^{\text{tree}} \left\{ -\frac{2}{\epsilon^2} \left(\frac{\mu^2}{-s_{12}} \right)^\epsilon - \frac{2}{\epsilon^2} \left(\frac{\mu^2}{-s_{51}} \right)^\epsilon - \frac{3}{\epsilon(1-2\epsilon)} \left(\frac{\mu^2}{-s_{51}} \right)^\epsilon - \delta_D \right\} - \frac{i}{(4\pi)^2} \sqrt{\frac{2}{[12]^2\langle 34\rangle^2\langle 25\rangle}} \\ & \times \left[2 \frac{\langle 14\rangle[12]^2\langle 25\rangle s_{34}(2[45]\langle 51\rangle\langle 24\rangle + 3\langle 23\rangle[34]\langle 14\rangle)}{\langle 45\rangle} R \ln^2 \left(\frac{s_{51}}{s_{23}} \right) - 4 \frac{\langle 34\rangle\langle 25\rangle s_{12}^2}{\langle 23\rangle\langle 45\rangle} (L(s_{51}, s_{45}, s_{23}) \right. \\ & \left. + L(s_{23}, s_{12}, s_{45})) + 4 \frac{\langle 25\rangle[51]\langle 34\rangle\langle 12\rangle s_{12}}{\langle 23\rangle\langle 24\rangle} L(s_{12}, s_{51}, s_{34}) \right. \\ & \left. - 2 \frac{\langle 25\rangle[12][45](\langle 14\rangle s_{51} s_{34} - \langle 24\rangle[25]\langle 51\rangle s_{45} s_{34})}{s_{51} s_{45} (s_{23} - s_{51})} + (3 \leftrightarrow 4) \right], \end{aligned}$$

where

$$\mathcal{M}^{\text{tree}} = \frac{2\sqrt{2}i\langle 12\rangle^2\langle 25\rangle}{\langle 45\rangle\langle 24\rangle\langle 23\rangle\langle 35\rangle}.$$

The tree-level amplitudes are given by

$$\mathcal{A}_5^{\text{tree}}(\bar{q}qg\gamma\gamma) = e^2 g T^a \mathcal{M}_5^{\text{tree}}(\bar{q}qg\gamma\gamma).$$

The amplitude with the helicity $(\bar{q}^-, q^+, \gamma^+, \gamma^-, g^+)$ is given by

$$\begin{aligned} \mathcal{M}_5^2(g_1^+ \bar{q}_2^- \gamma_3^- \gamma_4^+ q_5^+) = & -c_\Gamma \mathcal{M}^{\text{tree}} \left\{ -\frac{2}{\epsilon^2} \left(\frac{\mu^2}{-s_{12}} \right)^\epsilon - \frac{2}{\epsilon^2} \left(\frac{\mu^2}{-s_{51}} \right)^\epsilon - \frac{3}{\epsilon(1-2\epsilon)} \left(\frac{\mu^2}{-s_{51}} \right)^\epsilon - 2L(s_{34}, s_{23}, s_{51}) - 2L(s_{12}, s_{51}, s_{34}) \right. \\ & \left. - 2 - \delta_D \right\} - \frac{i}{(4\pi)^2} \sqrt{\frac{2}{[23]^2\langle 14\rangle^2\langle 25\rangle}} \left[f_1 R \ln^2 \left(\frac{s_{35}}{s_{12}} \right) + f_2 R \ln^2 \left(\frac{s_{35}}{s_{24}} \right) + f_3 R \ln^1 \left(\frac{s_{34}}{s_{12}} \right) \right. \\ & \left. + \left\{ -4 \frac{[45]\langle 51\rangle[12]\langle 24\rangle([45]\langle 51\rangle[12]\langle 24\rangle - \langle 23\rangle[34]\langle 45\rangle[25])^2}{s_{12} s_{45}^3} \right\} \right] \end{aligned}$$

$$\begin{aligned}
& -2 \frac{(2[45]\langle 51 \rangle [12]\langle 24 \rangle - [51]\langle 25 \rangle [24]\langle 14 \rangle - \langle 23 \rangle [34]\langle 45 \rangle [25])}{s_{45}^2} (2\langle 23 \rangle [34]\langle 45 \rangle [25] + 2[23]\langle 34 \rangle \\
& \times [45]\langle 25 \rangle + s_{12}s_{45}) + 2 \frac{[45](\langle 24 \rangle [12]\langle 51 \rangle - [23]\langle 34 \rangle \langle 25 \rangle) s_{51}s_{24}}{s_{45}^2} \\
& + 2 \frac{(-s_{51}s_{23} - 2s_{34}s_{23} + s_{51}^2 - s_{34}s_{51})s_{25}}{s_{45}} + 2 \frac{(\langle 45 \rangle [51]\langle 12 \rangle + \langle 23 \rangle \langle 14 \rangle [13])s_{23}^2}{s_{51}\langle 24 \rangle} \\
& - 2 \frac{[51](\langle 12 \rangle [23]\langle 35 \rangle + \langle 13 \rangle \langle 45 \rangle [34])(-s_{23} + s_{34})}{s_{51}} + 2 \frac{s_{23}^2(-s_{23} + s_{45})}{s_{51}} - 2[24](\langle 51 \rangle \langle 25 \rangle \langle 14 \rangle \\
& - \langle 34 \rangle \langle 12 \rangle [13]) + 2s_{51}^2 - 6s_{34}s_{51} + 2s_{51}s_{23} - 2s_{34}s_{45} - 4s_{51}s_{12} + 2s_{45}s_{23} + 4s_{34}^2 \\
& + 2s_{12}s_{34} \Big\} \tilde{L}(s_{34}, s_{35}, s_{12}) + 4 \frac{[14][25]s_{23}^2 L(s_{51}, s_{45}, s_{23})}{[12][45]} \\
& + \left\{ 2 \frac{[51](\langle 12 \rangle [23]\langle 35 \rangle + [34]\langle 45 \rangle \langle 13 \rangle) s_{34}^2 (s_{23} - s_{45})}{s_{51}s_{24}s_{14}} - 4 \frac{\langle 12 \rangle [13]\langle 34 \rangle^2 [23][45]\langle 25 \rangle}{\langle 24 \rangle s_{14}} \right. \\
& + 2 \frac{s_{34}(\langle 12 \rangle [13][24] - [23][45]\langle 25 \rangle)([12]\langle 23 \rangle \langle 14 \rangle + \langle 45 \rangle [51]\langle 13 \rangle)}{s_{24}s_{14}} \\
& \left. - 2 \frac{[34]^2([12]\langle 23 \rangle \langle 14 \rangle + \langle 45 \rangle [51]\langle 13 \rangle)^2}{s_{24}s_{14}} \right\} \tilde{L}(s_{24}, s_{12}, s_{35}) + R_0 \Big],
\end{aligned}$$

where

$$\mathcal{M}^{tree} = \frac{i2\sqrt{2}\langle 25 \rangle \langle 23 \rangle^2}{\langle 12 \rangle \langle 45 \rangle \langle 24 \rangle \langle 51 \rangle}.$$

We also used the following functions:

$$\begin{aligned}
\tilde{L}(s_{34}, s_{35}, s_{12}) &= L(s_{34}, s_{35}, s_{12}) - s_{45} R \ln^1 \left(\frac{s_{35}}{s_{12}} \right) + \frac{1}{2} s_{45}^2 R \ln^2 \left(\frac{s_{35}}{s_{12}} \right) - s_{45} R \ln^1 \left(\frac{s_{34}}{s_{12}} \right) \\
\tilde{L}(s_{24}, s_{12}, s_{35}) &= L(s_{24}, s_{12}, s_{35}) + \frac{1}{2} s_{14}s_{24} R \ln^2 \left(\frac{s_{35}}{s_{12}} \right).
\end{aligned}$$

f_i ($i=1,2,3$) and R_0 are

$$\begin{aligned}
f_1 &= -[24](\langle 45 \rangle [51]\langle 12 \rangle + [13]\langle 14 \rangle \langle 23 \rangle) s_{23}^2 \left(-\frac{-8s_{45}s_{23} + 3s_{23}^2 + 6s_{45}^2}{s_{51}s_{24}} - \frac{3s_{51} - 6s_{23} + 8s_{45}}{s_{24}} \right) \\
& - \frac{[51](\langle 13 \rangle \langle 45 \rangle [34] + \langle 12 \rangle [23]\langle 35 \rangle)(s_{23} - s_{45})(3s_{23}^2 - 3s_{34}s_{23} - 5s_{45}s_{23} + 3s_{34}^2 + 3s_{34}s_{45})}{s_{51}} \\
& + \frac{[12](\langle 45 \rangle \langle 51 \rangle \langle 24 \rangle - \langle 14 \rangle [34]\langle 23 \rangle) s_{23}^2 s_{45}}{s_{12}} + \frac{s_{23}^2 (s_{23} - s_{45})(-8s_{45}s_{23} + 3s_{23}^2 + 6s_{45}^2)}{s_{51}} + (2\langle 23 \rangle [34]\langle 45 \rangle [25] \\
& + 2\langle 23 \rangle [24]\langle 14 \rangle [13] - 2[51]\langle 25 \rangle [23]\langle 13 \rangle - s_{23}^2) s_{23} (2s_{51} + 4s_{45} - 5s_{23}) - 3([34]\langle 45 \rangle [51]\langle 13 \rangle + \langle 23 \rangle [24]\langle 14 \rangle [13] \\
& - 2\langle 12 \rangle [23]\langle 34 \rangle [14] + 2s_{34}s_{45} + s_{51}s_{23} - 2s_{23}^2)(-s_{45} + s_{12})(s_{45} + s_{34}) - s_{23}(s_{23}^3 - 6s_{34}s_{23}^2 - s_{23}^2 s_{12} - s_{51}s_{23}^2 + 2s_{23}^2 s_{45} \\
& - 2s_{51}s_{23}s_{12} - 7s_{23}s_{45}^2 - 2s_{23}s_{34}s_{45} + 3s_{23}s_{45}s_{12} + 6s_{34}^2 s_{45} - 2s_{12}s_{45}^2 + 2s_{45}^3 + 6s_{34}s_{45}^2)
\end{aligned}$$

$$f_2 = [51](\langle 13 \rangle \langle 45 \rangle [34] + \langle 12 \rangle [23] \langle 35 \rangle) s_{14} \left(-\frac{s_{23}(s_{23} - s_{45})}{s_{51}} + 3 \frac{s_{34}(s_{23} - s_{45})}{s_{51}} + 2 \frac{s_{12}(-s_{23} + s_{34})}{s_{51}} \right) + 3([23] \langle 34 \rangle [45] \langle 25 \rangle$$

$$- [51] \langle 12 \rangle [23] \langle 35 \rangle + s_{23}^2)(s_{12} - s_{45}) s_{14} + 2([23] \langle 34 \rangle [45] \langle 25 \rangle - [51] \langle 12 \rangle [23] \langle 35 \rangle - s_{34} s_{12})(s_{12} + s_{23}) s_{14} + 2[14](\langle 12 \rangle$$

$$\times [23] \langle 34 \rangle + \langle 51 \rangle [25] \langle 24 \rangle) s_{12} s_{25} - 2 s_{14} s_{23}^2 (-s_{23} + s_{51} + s_{12} + s_{34}) - s_{14} (-2 s_{51} + 2 s_{34} + 3 s_{23}) s_{12}^2 - s_{14} (3 s_{34} - s_{23}) s_{45}^2$$

$$+ s_{14} s_{34} (-4 s_{51} + 2 s_{34} + 5 s_{45}) s_{12} + 2 s_{14} (3 s_{23} s_{34} s_{45} + s_{51}^2 s_{12})$$

$$f_3 = -\frac{[45](\langle 51 \rangle [12] \langle 24 \rangle - [23] \langle 34 \rangle \langle 25 \rangle) s_{23} (s_{34} s_{23} - s_{12} s_{23} - 2 s_{51} s_{12})}{s_{45} s_{12}}$$

$$+ \frac{([51] \langle 25 \rangle [23] \langle 13 \rangle + \langle 23 \rangle [34] \langle 45 \rangle [25]) s_{23} (3 s_{25} s_{25} + 2 (s_{51} - s_{34}) s_{45})}{s_{51} s_{24}} - \frac{s_{45} s_{23}^3 (3 s_{23} + 3 s_{12} - 2 s_{45})}{s_{51} s_{24}}$$

$$- 2 \frac{s_{23} (-s_{23} - s_{51} + s_{34}) s_{45}^2}{s_{51}} + 3 \frac{s_{45} s_{23}^2 (s_{25} - s_{23})}{s_{51}} + 3 \frac{(\langle 23 \rangle [34] \langle 45 \rangle [25] - \langle 23 \rangle [24] \langle 14 \rangle [13] - s_{12}^2) s_{23}^2}{s_{24}}$$

$$- \frac{s_{23}^2 (6 s_{12} s_{23} - 2 s_{45} s_{12} - 5 s_{45} s_{23} + 3 s_{23}^2)}{s_{24}} - \frac{s_{23}^2 s_{34} (-s_{51} + s_{34})}{s_{12}} + 4 (-\langle 23 \rangle [24] \langle 14 \rangle [13] + 2 [51] \langle 25 \rangle [23] \langle 13 \rangle$$

$$+ [12] \langle 23 \rangle [34] \langle 14 \rangle) s_{23}$$

$$R_0 = (\langle 51 \rangle [12] \langle 23 \rangle [35] - \langle 45 \rangle [51] \langle 12 \rangle [24] + s_{12} s_{35} - s_{51} s_{35}) \left(\frac{s_{12}(s_{51} + s_{45})}{s_{35}(s_{12} + s_{14})} - \frac{s_{12}(-s_{51} + s_{12})}{s_{51}(s_{12} + s_{14})} \right) + (-\langle 45 \rangle [51] \langle 12 \rangle [24]$$

$$- \langle 23 \rangle \langle 14 \rangle [13] [24] + s_{14}^2) \left(-\frac{s_{14} s_{12} (s_{14} + s_{45})}{s_{45} s_{35} (s_{12} - s_{35})} - \frac{s_{14} (s_{51} + s_{45})}{s_{35} (s_{12} - s_{35})} \right)$$

$$- \frac{(\langle 45 \rangle [51] \langle 12 \rangle [24] + [12] \langle 23 \rangle [34] \langle 14 \rangle) ((s_{51} - s_{23})^2 - s_{45} s_{14})}{s_{45} s_{35}} + 2 \frac{s_{14}^2 s_{45} (s_{51} + s_{45})}{s_{35} (-s_{12} + s_{35})}$$

$$- \frac{s_{14}^2 s_{51} (s_{45} s_{12} - s_{45}^2 - s_{51} s_{45} - s_{14} s_{12})}{s_{45} s_{35} (-s_{12} + s_{35})} - 2 \frac{s_{51}^3 s_{12}}{s_{45} s_{35}} + 2 \frac{[24](\langle 45 \rangle [51] \langle 12 \rangle + \langle 25 \rangle [35] \langle 34 \rangle) (-s_{34} + s_{51})^2}{s_{51} s_{24}}$$

$$- 2 \frac{s_{12}^2 (-s_{45} s_{35} + s_{51}^2 + s_{51} s_{45})}{s_{35} (s_{12} + s_{14})} + \frac{(s_{45} s_{12} - s_{23}^2) s_{14}}{s_{35}} + \frac{([45] \langle 51 \rangle [12] \langle 24 \rangle + [51] \langle 14 \rangle [34] \langle 35 \rangle + s_{51}^2 - s_{23}^2) s_{51}}{s_{35}}$$

$$- 2 \frac{(\langle 45 \rangle [51] \langle 12 \rangle [24] + [12] \langle 23 \rangle [34] \langle 14 \rangle) s_{23}^2}{s_{45} s_{12}} + \frac{s_{23} (s_{51}^2 + s_{23}^2)}{s_{45}} - 2 \frac{s_{45} (s_{12} s_{23} - s_{34}^2)}{s_{51}}$$

$$+ \frac{(\langle 51 \rangle [12] \langle 23 \rangle [35] + \langle 12 \rangle [13] \langle 35 \rangle [25] + s_{45}^2) (-s_{23} + s_{45} + s_{12})}{s_{51}} - \frac{s_{45}^2 (s_{23} + s_{34})}{s_{51}}$$

$$- 2 \frac{(-[51] \langle 14 \rangle [34] \langle 35 \rangle + \langle 34 \rangle [35] \langle 25 \rangle [24] - s_{34} s_{45}) (s_{23} - s_{34})}{s_{51}} - \frac{-s_{34} s_{23}^2 + s_{23}^2 s_{12}}{s_{51}} + \frac{s_{45}^3 (s_{13} + s_{24})}{s_{51} s_{35}}$$

$$- 7 \frac{s_{34} s_{45} (s_{51} + s_{45})}{s_{35}} - 6 \frac{s_{23} s_{45}^2 + s_{51}^2 s_{34}}{s_{35}} - 4 \frac{s_{51} s_{45} (s_{23} + s_{34})}{s_{35}} - 2 [12] \langle 23 \rangle [34] \langle 14 \rangle - 2 \langle 34 \rangle [35] \langle 25 \rangle [24] - 4 s_{34} s_{45}$$

$$+ 2 s_{14} s_{51} - 2 s_{23}^2 + 4 s_{34} s_{23} - s_{45}^2 - 4 s_{51}^2 - 2 s_{34}^2 - s_{45} s_{12} - s_{51} s_{12} - s_{12} s_{23} - 3 s_{51} s_{45} - s_{45} s_{23} + s_{34} s_{12}.$$

C. $\mathcal{M}_5^3(\bar{q} q g \gamma \gamma)$

In the end of this section, we give amplitudes which come from the fermion loop contributions. Here, we only consider massless quark loops:

$$\mathcal{M}_5^3(q_1^+ \bar{q}_2^- g_3^+ \gamma_4^+ \gamma_5^+) = \frac{16}{i\sqrt{2}(4\pi)^2} \frac{\langle 45 \rangle [51] \langle 12 \rangle [24] s_{14} - \langle 45 \rangle [51] \langle 12 \rangle [24] s_{24} - s_{25} s_{24} s_{14} + s_{24}^2 s_{51}}{s_{12} [24] \langle 41 \rangle \langle 35 \rangle \langle 54 \rangle \langle 43 \rangle}$$

$$\begin{aligned}
\mathcal{M}_5^3(q_1^+ \bar{q}_2^- g_3^+ \gamma_4^- \gamma_5^+) &= \mathcal{M}_5^3(q_1^+ \bar{q}_2^- \gamma_3^+ g_4^- \gamma_5^+) = \frac{1}{i(4\pi)^2} \frac{8}{\sqrt{2}[24]^2 \langle 35 \rangle^2 \langle 12 \rangle} \left[-2 s_{24}^2 (-s_{12} - s_{35}) s_{35} R \ln^2 \left(\frac{s_{34}}{s_{12}} \right) \right. \\
&+ \left\{ 2 \frac{(-\langle 51 \rangle [12] \langle 23 \rangle [35] - \langle 45 \rangle [14] \langle 13 \rangle [35] + s_{23} s_{12}) s_{14}}{s_{35}} \right. \\
&+ 2 \frac{(\langle 12 \rangle [23] \langle 34 \rangle [14] - \langle 51 \rangle [12] \langle 23 \rangle [35]) s_{12} s_{23}}{(-s_{12} + s_{45}) s_{35}} + 2 \frac{s_{34}^2 (-s_{23}^2 + s_{12} s_{51} + s_{23} s_{51})}{-s_{12} + s_{45}} \Big)^2 \\
&- \frac{(\langle 12 \rangle [13] \langle 34 \rangle [24] - \langle 51 \rangle [12] \langle 23 \rangle [35] - s_{34} s_{23} + 2 s_{34} s_{51}) (s_{34} s_{12} + s_{34} s_{23} + s_{23} s_{12})}{(-s_{12} + s_{45})^2} \\
&- \frac{(\langle 23 \rangle [34] \langle 45 \rangle [25] + \langle 12 \rangle [23] \langle 34 \rangle [14]) (-s_{34} + s_{51} + 2 s_{23})}{-s_{12} + s_{45}} \\
&- 2 \frac{[13] (\langle 12 \rangle \langle 34 \rangle [24] - \langle 23 \rangle [25] \langle 51 \rangle) s_{12}}{-s_{12} + s_{45}} - \frac{s_{23} (-s_{34}^2 + s_{34} s_{51} + 2 s_{12}^2 + 4 s_{12} s_{51})}{-s_{12} + s_{45}} \\
&+ 2 [34] \langle 35 \rangle [51] \langle 14 \rangle + 2 \langle 45 \rangle [14] \langle 13 \rangle [35] + 2 [45] \langle 14 \rangle [12] \langle 25 \rangle \\
&- 2 \langle 23 \rangle [24] \langle 45 \rangle [35] - 4 s_{23} s_{51} + 4 s_{34} s_{23} - 4 s_{23} s_{12} + 2 s_{51} s_{45} + s_{51}^2 + s_{34}^2 - 2 s_{34} s_{12} \Big\} \\
&\times \left\{ \ln \left(\frac{s_{45}}{s_{12}} \right) - \frac{(s_{45} - s_{12})^2}{(s_{34} - s_{12})^2} \ln \left(\frac{s_{34}}{s_{12}} \right) \right\} - 2 [35]^2 [24]^2 (\langle 34 \rangle^2 \langle 25 \rangle^2 + \langle 23 \rangle^2 \langle 45 \rangle^2) \\
&\times \left\{ \frac{L(s_{34}, s_{45}, s_{12})}{s_{35}^2} - \frac{(2s_{45} - 3s_{12} + 2s_{34})}{s_{35}} R \ln^2 \left(\frac{34}{12} \right) \right\} \\
&\times \frac{(\langle 23 \rangle [34] \langle 45 \rangle [25] + \langle 12 \rangle [23] \langle 34 \rangle [14]) s_{23} s_{34}}{s_{12} (-s_{12} + s_{45})} \\
&- \frac{(\langle 34 \rangle [45] \langle 51 \rangle [13] + \langle 45 \rangle [51] \langle 12 \rangle [24]) s_{45} s_{51}}{s_{12} (s_{34} - s_{12})} \\
&\times \frac{(\langle 12 \rangle [13] \langle 34 \rangle [24] - \langle 51 \rangle [12] \langle 23 \rangle [35]) (s_{34} + s_{23})}{-s_{12} + s_{45}} + 2 \frac{s_{23} s_{51} s_{34}}{-s_{12} + s_{45}} \\
&+ \frac{(-\langle 45 \rangle [51] \langle 12 \rangle [24] - \langle 51 \rangle [12] \langle 23 \rangle [35] - s_{12} s_{45}) (-s_{45} + s_{51})}{s_{34} - s_{12}} \\
&+ 2 \frac{s_{51} s_{45} (-s_{45} + s_{23} + s_{12})}{s_{34} - s_{12}} + \frac{(s_{45} + s_{34}) (-s_{45} + s_{23}) (-s_{34} + s_{51})}{s_{12}} \\
&+ \frac{(\langle 12 \rangle [23] \langle 34 \rangle [14] - \langle 45 \rangle [51] \langle 12 \rangle [24] + s_{45} s_{34}) (-s_{45} - s_{34} + s_{23} + s_{51})}{s_{12}} \\
&+ s_{34}^2 - 2 s_{34} s_{51} + s_{45}^2 + 4 s_{34} s_{23} + s_{51}^2 + s_{23}^2.
\end{aligned}$$

Now, we give some comments concerning the cross-check of our results. Besides the previous results, we also carried out the direct calculation of the amplitudes with the helicity configurations $(\bar{q}^-, q^+, g^+, \gamma^-, \gamma^\pm)$ and $(\bar{q}^-, q^+, g^-, \gamma^-, \gamma^\pm)$. These amplitudes are also obtained from the previous results by using parity inversion and charge conjugation. This procedure is basically the same procedure discussed in Sec. II. Our direct calculations are consistent with this argument.

In addition, we checked the singular parts. Singular parts of one-loop amplitudes have the well-known universal structure Eq. (8) [18,19]. Thus, we can easily compute the singular part of the amplitudes from the known results of Ref. [11] by using the procedure given in Refs. [9,10]. To evaluate the singular part, we have to keep in mind that the regularization scheme does not affect the universal structure of the $1/\epsilon^2$ pole parts but the $1/\epsilon$ pole parts are scheme dependent. We estimated the singular parts in the FDH scheme and our results are consistent with the results of Refs. [9,10].

We also performed a consistency check of the scheme dependence. It is well known that the FDH scheme at the one-loop level is equivalent to the dimensional reduction (DR) scheme [21]. The conversion rule between the DR scheme and the 't Hooft–Veltman scheme for the two-quark $(n-2)$ gluon amplitudes is discussed in Refs. [11,22]. From their argument, the conversion from the DR scheme to the 't Hooft–Veltman scheme at the one-loop level is obtained by shifting the amplitudes as $A_n \rightarrow A_n + \delta_n$ with $\delta_n = -c_F(1 - 1/N_c^2)A_n^{tree}$. Now, we can consider the colorless limits

($C_F \rightarrow 1$ and $C_A \rightarrow 0$) of our results and the results in Refs. [10,11]. In these limits, both results give the same quantity of shifting parameter δ_5 .

IV. CONCLUSION

In this paper, we presented one-loop five-parton amplitudes involving two massless quarks and two- or three-photon external legs. The one-loop five parton amplitudes with external photons are required to evaluate the QCD background for the Higgs boson production with a jet. These amplitudes have been discussed [9,10] in terms of the known QCD amplitude ($q\bar{q}ggg$) [11]. They give the systematic procedure to replace external gluons into photons. However, their results are still in a rather large expression. Here, we computed the one-loop helicity amplitudes for the process $q\bar{q}g\gamma\gamma$ directly and successfully obtained a more compact expression. With the amplitudes which we presented here, together with the six parton tree-level amplitudes ($q\bar{q}\gamma\gamma gg$, $q\bar{q}Q\bar{Q}\gamma\gamma$) [10], we can estimate the NLO QCD background for the associated Higgs boson production with a jet.

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